Supplementary Information

PVDF Microbelts for Harvesting Energy from Respiration

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S1. Simulation of Vibration

To simulate the vibration of PVDF microbelt (MB) subject to laminar flow, following governing equation is applied,[1]

\[
\frac{\partial^2}{\partial z^2} (EI \frac{\partial^2 Y(z, t)}{\partial z^2}) + m \frac{\partial^2 Y(z, t)}{\partial t^2} = F(z, t)
\]

With separation of variables,

\[ Y(z, t) = y(t) \tilde{y}(z) \]

Then, \( y(t) \) can be written as,

\[ \ddot{y} + 2\zeta \omega y + \omega^2 y = \sum_{i=1}^{\infty} \beta_i a_i \dot{y}' \]

\[ \beta_i = \int_0^L u^{2-i}(z) \tilde{y}'^{(i)}(z) dz \bigg/ \int_0^L \tilde{y}^2(z) dz \]

where \( y(t) \) is the vertical displacement at time \( t \), \( \tilde{y}(z) \) and \( \omega \) are the oscillation mode and frequency of the beam, respectively \( (\omega = (\pi^2 / L^2) \sqrt{EI/m}, \) E the Young’s modulus, I the moment of inertial), \( u(z) \) the flow speed, \( \zeta \) damping factor, \( m \) mass per unit length, \( F \) applied force and \( a_i \) are a series of aerodynamic coefficients.

With both ends of the PVDF MB clamped, the boundary conditions are obtained,

\[ Y(0, t) = Y(L, t) = 0, Y'(0, t) = Y'(L, t) = 0. \]

Then we have,
And \( y(t) \) was numerically solved by Runge-Kutta method.

\[
\ddot{y}_n(z) = \cosh\left(\frac{(2n+1)\pi z}{2L}\right) - \cos\left(\frac{(2n+1)\pi z}{2L}\right) - A_n \left(\sinh\left(\frac{(2n+1)\pi z}{2L}\right) - \sin\left(\frac{(2n+1)\pi z}{2L}\right)\right)
\]

Figure S1. Simulated oscillation of a PVDF MB excited by a constant disturbance. Inset is the schematic cross-section geometry of the MB.

Figure S1 shows the simulated oscillation of a PVDF microbelt with dimension of 20 \( \mu \text{m} \) (thickness) \( \times 2 \text{ cm} \) (length) \( \times 2 \text{ mm} \) (width) under wind speed of 1.4 m/s. Here, density of PVDF is 1780 kg/m\(^3\); Young modulus is 1.1 GPa; damping factor \( \zeta \) is 0.0005. Aerodynamic coefficients used in the calculation are experimental values acquired from large-scale models, where \( a_0, a_1, a_2, a_3, \) and \( a_4 \) are 0, 1.9142, 34.789, -170.97 and -22.074, respectively. The resonant frequency is calculated to be 252 Hz.
S2. Calculation of Voltage and Power Output

To predict the output voltage, in details, shape and amplitude of the PVDF MB at its maximum oscillation amplitude were acquired first based on the procedure described in S1, as shown in Fig. S2.

![Figure S2. Shape and amplitude of PVDF microbelt subject to laminar flow. Dimension of belt and flow speed are same as the one in Fig. S1.](image)

Second, the overall length of the PVDF MB at its maximum oscillation amplitude ($L'$) was obtain by integration shape curve shown in Fig. S2. Thus, the maximum strain could be extracted,

$$\varepsilon = \frac{L' - L}{L}$$

where $L$ is the original length of the PVDF MB. To be noticed, when the strain was larger than 5%, we assume the film reaches its maximum tolerable strain.$^{[2]}$

The voltage is approximately estimated by,

$$V = \frac{Q}{C} = \frac{DA}{C} = \frac{d_{31} \varepsilon E A}{\varepsilon_r \varepsilon_0 \frac{A}{d}} = \frac{d_{31} \varepsilon E d}{\varepsilon_r \varepsilon_0 A}$$

where $Q$ is the generated charge upon deformation, $C$ is capacitance, $D$ is charge displacement, $A$ is the electrode area, $E$ is Young’s modulus, $\varepsilon$ is strain, $\varepsilon_r$ and $\varepsilon_0$ are relative permittivity (12.4)$^{[3]}$ and electric constant, respectively, $d_{31}$ is piezoelectric coefficient (16 pC/N)$^{[3]}$ and $d$ is the thickness of belt.

Based on this algorithm, the maximum possible open circuit voltages were calculated as functions of air flow speeds from 0.2 m/s to 3 m/s and PVDF film thickness from 0.1 µm to 45 µm. In the calculation, the length and width of the PVDF MB were kept as constants which are 2 nm and 2 mm, respectively. This relationship was plotted as shown in Figure S3.
Figure S3. Plot of the calculated maximum possible open circuit voltages that can be produced by a PVDF film under different air flow speed and film thickness.

With the calculated maximum open circuit voltage output, corresponding electric power was also calculated following the procedure desribed in the main content. The power versus film thickness and wind speed was plotted, as shown in Figure S4.

Figure S4. Plot of the calculated maximum electric power output of a PVDF film under different air flow speed and film thickness.
S3. Determine the Maximum Output Power

Figure S5. Measured output voltage with different load resistance (blue dots and line) attached to the PVDF MB. The output power was calculated by substitute the voltage value and corresponding resistance to equation (1) in the main content (red dots and line), from which the maximum output power can be determined.

S4. Movie1: Operation of an electronic stopwatch powered by an oscillating PVDF MB.

References: